## applied optics

# TWINS: improved spatial and angular phase calibration for holography: supplement

ALEXEY SUPIKOV,  $^{1,*}$  MICHAEL MEFENZA NENTEDEM,  $^1$  ANDERS GRUNNET-JEPSEN,  $^{1,2}$  AND RONALD T. AZUMA  $^1$ 

This supplement published with Optica Publishing Group on 9 October 2023 by The Authors under the terms of the Creative Commons Attribution 4.0 License in the format provided by the authors and unedited. Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI.

Supplement DOI: https://doi.org/10.6084/m9.figshare.24178404

Parent Article DOI: https://doi.org/10.1364/AO.499387

<sup>&</sup>lt;sup>1</sup>Intel Labs, Intel Corporation, 2200 Mission College Blvd, Santa Clara, California 95054, USA

<sup>&</sup>lt;sup>2</sup>Currently with Luminar Technologies, Orlando, Florida 32826, USA

<sup>\*</sup>alexei.soupikov@intel.com

### TWINS: IMPROVED SPATIAL AND ANGULAR PHASE CALIBRATION FOR HOLOGRAPHY: SUPPLEMENTAL DOCUMENT

#### 1. Two physical slits with phase difference

Given two parallel narrow slits of width a separated by distance d we can extend the far-field diffraction integral to describe the more general case when the light leaving each slit has different constant phase shifts  $\varphi_1$  and  $\varphi_2$  by taking the Fourier transform of the optical field at source plane coordinates  $x_0$ , computed at scaled target plane positions  $x_1$ .

The equality (S1) can be shown by using linearity, shift property and *rect* properties of the Fourier transform, followed by a sum of 2 complex numbers in exponential form [1].

$$\mathcal{F}\left(rect\left(\frac{x_0}{a} - \frac{d}{2}\right)exp(i\varphi_1) + rect\left(\frac{x_0}{a} + \frac{d}{2}\right)exp(i\varphi_2)\right)\left(\frac{x_1}{\lambda z}\right) = asinc\left(\frac{\pi a x_1}{\lambda z}\right)exp\left(i\frac{\varphi_1 + \varphi_2}{2}\right)$$

$$\sqrt{2 + 2\cos\left(\varphi_1 - \varphi_2 - \frac{2\pi d x_1}{\lambda z}\right)} \tag{S1}$$

Squaring the amplitude results in formula (1) of the main paper.

#### 2. Possible hardware setups

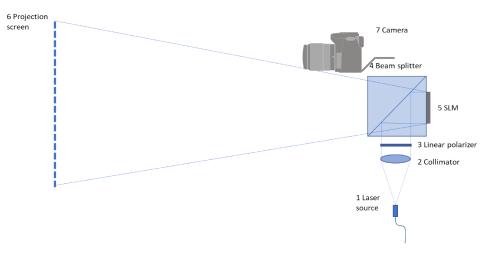
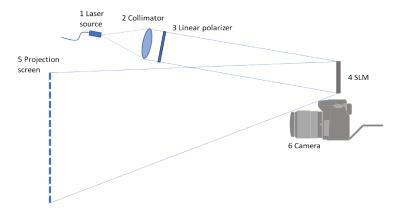


Fig. S1. On-axis illumination (not up to scale)



#### 3. Test image examples

High contrast images with sharp details are the most challenging for far field holography so we added synthetic images representing typical content of an automotive head-up display (HUD). The HUD images were automatically generated by adding HUD elements to base images. Fig. S3 focuses on the artifacts of using a perfect "isotropic" hologram with anisotropic phase modulation and shows how an "anisotropic" hologram fixes it.

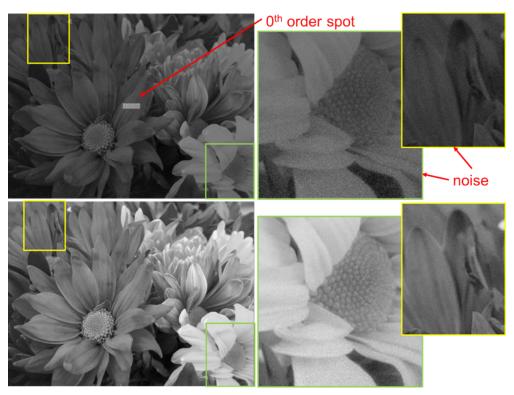


Fig. S3. Nature 1, anisotropic phase, same beam intensity. Top: regular Wirtinger hologram (note brightness loss and noise at top and bottom). Bottom: our anisotropic hologram.

Note that all isotropic holograms have reduced brightness as much of the energy goes to the bright 0th-order spot due to diffraction efficiency loss. Isotropic images are also very noisy at the bottom and top areas, where the modulation range highly deviates from the nominal 0 to  $2\pi$  range. Anisotropic holograms fix the energy distribution problem and mostly address noise problems.

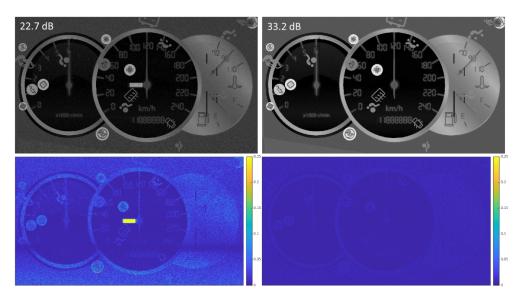


Fig. S4. Isotropic (left) and anisotropic (right) hologram (top) of image HUD 20 on anisotropic LCoS with absolute error maps (bottom)

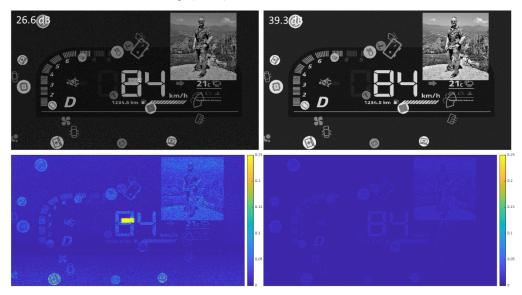


Fig. S5. Isotropic (left) and anisotropic (right) hologram (top) of image HUD 28 on anisotropic LCoS with absolute error maps (bottom)



Fig. S6. Isotropic hologram of image Nature 1 on anisotropic LCoS (low brightness, noisy)



Fig. S7. Anisotropic (new) hologram of image Nature 1 on anisotropic LCoS



Fig. S8. Isotropic hologram of image HUD 28 on anisotropic LCoS (low brightness, noisy)



Fig. S9. Anisotropic (new) hologram of image HUD 28 on anisotropic LCoS

#### 4. Crosstalk and static phase aberrations

Since slits are small and propagate very little energy, measurements might be vulnerable to cross talk. Once all reflective surfaces are covered with light absorbing material, the major source of crosstalk is static phase aberration, which creates a "cross" pattern that extends vertically and horizontally, into the captured fringe pattern. If the slit grating period is large so the fringe pattern is close to the center, then large phase aberrations can substantially reduce measurement accuracy, which can be addressed by increasing the dimensions of the slits.

To evaluate the effect of crosstalk, we simulated a measurement process using two different phase curvature profiles captured using [2] with different focus positions of the collimator. Measurements were collected at 2 locations, one closer to the SLM center, and the other closer to one of the corners. Please refer to Fig. S10 and Fig. S11 for the details.

We used slits of period 2 (the smallest possible) and of period 6 (close to the center). Double slits of 2 sizes were used for our simulation. Small slits: width 4, spacing 12, length 120 pixels. Large slits: width 8, spacing 12, length 180 pixels. The results are presented at table S1. In all our experiments the absolute error increases with the gray level, reaching its maximum around  $2\pi$ , so the relative error stays small.

Increasing slit dimensions to use more energy helps remedy the problem, albeit at the cost of reducing spatial resolution.

	Small aberrations		Large aberrations	
Slits configuration	Location 1	Location 2	Location1	Location 2
Small slits, period 2	0.0068	0.0096	0.0156	0.0169
Small slits, period 6	0.0581	0.0686	0.1435	0.1574
Large slits, period 2	0.0011	0.0092	0.008	0.0101
Large slits, period 6	0.0070	0.0165	0.0287	0.0352

Table S1. Maximum phase measurement error, rad

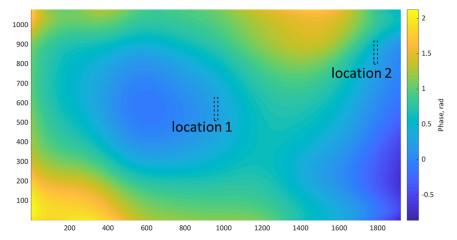


Fig. S10. Per-pixel map of small phase aberrations and double slit locations

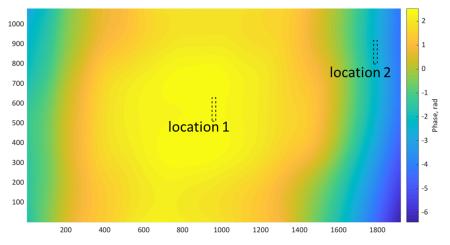


Fig. S11. Per-pixel map of large phase aberrations and double slit locations

#### References

- "Sum of Complex Numbers in Exponential Form", <a href="https://proofwiki.org/wiki/Sum\_of\_Complex\_Numbers\_in\_Exponential\_Form">https://proofwiki.org/wiki/Sum\_of\_Complex\_Numbers\_in\_Exponential\_Form</a>
  Jan Bolek and Michal Makowski, "Non-invasive correction of thermally induced wavefront aberrations of spatial light modulator in holographic projection," Opt. Express 27, 10193-10207 (2019)